Homework 2

Instructions: Legibly complete each of the following on lined paper and submit on Gradescope.

$\boxed{*}$ 1. Let $A = \{1, 3, 5\}$, $B = \{-1, 1, 2, 4\}$, and $C = \{2, 4, 6\}$. Compute each of the following.

(a) $A \cap B$

(b) $A \cup C$

(c) $B \setminus C$

(d) $B \times A$

(e) $\mathbb{P}(A)$

2. Let A, B, and C be sets. Prove the following (via any valid method).

(a) $A \cup B = B \cup A$

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(c) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

$\boxed{*}$ (d) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$\boxed{*}$ (e) $\mathbb{P}(A) \cup \mathbb{P}(B) \subseteq \mathbb{P}(A \cup B)$

$\boxed{*}$ 3. Prove or disprove: there exist sets A_1, A_2, A_3 such that $A_i \cap A_j \neq \emptyset$ for all $1 \leq i, j \leq 3$ but $A_1 \cap A_2 \cap A_3 = \emptyset$.

4. Let A, B, and C be arbitrary sets. Prove $(x, x) \in A \times B$ if and only if $x \in A \cap B$.

$\boxed{*}$ 5. Let $a, b, c \in \mathbb{Z}$ be arbitrary. Prove that $a \mid b$ and $a \mid c$ implies $a \mid (bs + ct)$ for all $s, t \in \mathbb{Z}$.

$\boxed{*}$ 6. Let $m, n \in \mathbb{Z}$ with $m \neq 0$. Prove that if $n = mq + r$ for some $q, r \in \mathbb{Z}$, then $\gcd(m, n) = \gcd(m, r)$.